

A Rocky Mountain Chapter White Paper: “The Mathematics of Field Strength Measurements”

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The following is adapted from an article I wrote about the mathematics behind signal leakage field strength measurements, in particular the origin of the “0.021” factor used when converting between microvolts and microvolts per meter. That article originally appeared in the July 1991 issue of *Communications Technology*, with an update published in June of 2008.

The original 1991 article that is the basis for this installment of *A Rocky Mountain Chapter White Paper* was inspired by a call from long-time industry friend and colleague Ralph Haimowitz, who commented that he had seen a recent paper discussing, among other things, the conversion between microvolts (μV) and microvolts per meter ($\mu\text{V}/\text{m}$). The paper used a somewhat different multiplier than the familiar “0.021” that’s in the formulas we still use today! Ralph’s concern was the effect the multiplier used in the paper would have on leakage measurement calculations performed by cable operators. So I sat down with a variety of references and did some number crunching related to the math of field strength measurements. Grab a cup of hot coffee and join me as we revisit where “0.021” comes from and why it is indeed “0.021.”

When we measure signal leakage, we’re dealing with a source of RF power (the leak), its propagation from the source to a location where the measurement is made, and determination of the radio frequency (RF) signal’s field strength at the point of measurement. The receive antenna is normally a half-wave dipole that is resonant at the frequency of interest. In fact, Federal Communications Commission (FCC) rules require that signal leakage measurements be made with a dipole, or at least correlated to a dipole measurement.

Most discussions of RF propagation and field strength measurements reference an isotropic radiator or antenna. This is a theoretical and ideal point source that radiates equally in all directions and has unity gain. Even though an isotropic antenna does not actually exist, it’s useful as a mathematical basis against which real-world comparisons can be made.

Imagine an isotropic antenna in free space, radiating a certain amount of RF power (P_t) uniformly in all directions. Now imagine a sphere whose surface is some distance from the point of radiation, with that point (the isotropic antenna) at the center of the sphere. A good analogy here is a very tiny but bright light bulb – an “isotropic radiator” – in the center of a large balloon, with the bulb illuminating the balloon from the inside. You could define the amount of power in a given area at the surface of the sphere (or balloon) – say, in a square meter – as so many watts per unit of area. Indeed, that’s exactly how RF power density is characterized.

In this example, the power density P_d (in watts per square meter) at the surface of the sphere is simply the total transmitted power P_t divided by the surface area of the sphere ($4\pi R^2$):

$$P_d = P_t/4\pi R^2 \quad (\text{Eq 1})$$

The level of the radiated RF power also can be expressed in volts per meter (V/m). If the field strength on our imaginary sphere has an intensity of E in V/m, then the power density is:

$$P_d = E^2/120\pi \quad (\text{Eq 2})$$

You'll notice that Equation 2 is an expression of the basic power equation with Ohm's Law equivalents substituted for E and I , or $P = E^2/R$. In this case, 120π is the "resistance," or impedance of free space (approximately 377 ohms). In order for power density to be useful, though, it has to be converted to received power. To do that, multiply power density in watts per square meter by the area in square meters of the receiving antenna. Keep in mind there is little relationship between the actual size of a dipole antenna and its *effective* area. If you know the numerical gain (g) of an antenna you can calculate its effective area (A_e) with the formula:

$$A_e = g\lambda^2/4\pi \quad (\text{Eq 3})$$

For a half-wave dipole, the effective area is:

$$A_e = 1.64\lambda^2/4\pi \quad (\text{Eq 4})$$

Note that in Equation 4, 1.64 is the numerical gain of a half-wave dipole in free space. Its gain in decibels relative to an isotropic antenna is $10\log(1.64) = 2.15$ dBi.

As mentioned previously, the power intercepted by a receiving antenna is found by multiplying the antenna's effective area by the power density at the point of measurement:

$$P_r = A_e P_d \quad (\text{Eq 5})$$

Substituting Equations 2 and 4 for P_d and A_e in Equation 5:

$$P_r = 1.64\lambda^2 E^2/480\pi^2 \quad (\text{Eq 6})$$

We can now calculate the received voltage at the terminals of a half-wave dipole antenna using the power equation variation of Ohm's Law:

$$E_r = \sqrt{P_r Z} \quad (\text{Eq 7})$$

where P_r is the received power (from Equation 6) and Z is the impedance of a half-wave dipole in free space. The latter is $Z = 73.13 + j42.5$ ohms for an infinitely thin dipole in

free space, but we'll assume no reactance and call the dipole's impedance approximately 73.2 ohms. Substituting Equation 6 for P_r and 73.2 for Z , Equation 7 becomes:

$$E_r = \sqrt{(1.64\lambda^2 E^2 / 480\pi^2) 73.2} \quad (\text{Eq 8})$$

The free space wavelength of an RF signal can be found with the formula:

$$\lambda = c / f_{\text{Hz}} \quad (\text{Eq 9})$$

where c is the speed of light (299,792,458 meters per second) and f_{Hz} is the frequency in hertz. When dealing with frequencies in megahertz the formula becomes:

$$\lambda = 299.792458 / f_{\text{MHz}} \quad (\text{Eq 10})$$

Substituting Equation 10 for λ in Equation 8, a little algebra will reduce Equation 8 to a simpler form:

$$E_r = 47.72299333 E / f_{\text{MHz}} \quad (\text{Eq 11})$$

With Equation 11 we can calculate the received voltage (in volts) from a half-wave dipole in free space when we know the field strength in V/m (E) and the frequency in MHz (f_{MHz}). If you want to change units so that the received voltage is in μV and the field strength is in $\mu\text{V}/\text{m}$, the same equation is used. Just make sure you use the same units; don't mix units – for example volts for E_r and $\mu\text{V}/\text{m}$ for E .

To convert a received voltage in μV to dBmV, use:

$$\text{dBmV} = 20 \log(E_r / 1,000) \quad (\text{Eq 12})$$

This can be used to modify Equation 11:

$$E_r = 20 \log(0.04772299333 E / f_{\text{MHz}}) \quad (\text{Eq 13})$$

E_r is the received voltage in μV from the dipole, and E is the field strength in $\mu\text{V}/\text{m}$. Going the other way, you can determine the field strength in $\mu\text{V}/\text{m}$ if you know the frequency in MHz and the dipole's received signal level in dBmV:

$$\mu\text{V}/\text{m} = 20.9543 \times f_{\text{MHz}} \times 10^{(\text{dBmV}/20)} \quad (\text{Eq 14})$$

Here 20.9543 is the reciprocal of 0.04772299333 from Equation 13. If you divide by 1,000, you'll have the familiar 0.021 (actually 0.0209543) multiplication factor used in the more common conversion from μV to $\mu\text{V}/\text{m}$:

$$\mu\text{V}/\text{m} = \mu\text{V} \times 0.021 \times f_{\text{MHz}} \quad (\text{Eq 15})$$

dBmV can be converted to microvolts with the formula:

$$\mu\text{V} = 10^{(\text{dBmV}/20)} \times 1,000 \quad (\text{Eq 16})$$

Thus, when you know the received signal level in dBmV, the conversion to $\mu\text{V}/\text{m}$ becomes a slight variation of Equation 14:

$$\mu\text{V}/\text{m} = 10^{(\text{dBmV}/20)} \times 1,000 \times 0.021 \times f_{\text{MHz}} \quad (\text{Eq 17})$$

Now let's put this to work. Assume that you've measured a leaking continuous wave (CW) carrier on Ch. 14's visual carrier frequency (121.2625 MHz) with a half-wave dipole held 3 meters (approx. 10 feet) from the plant, and the leak's level at the dipole terminals is -44.6 dBmV. What's the field strength in $\mu\text{V}/\text{m}$? Use Equation 17:

$$\begin{aligned} \mu\text{V}/\text{m} &= 10^{(-44.6/20)} \times 1,000 \times 0.021 \times 121.2625 \\ &= 10^{-2.23} \times 1,000 \times 0.021 \times 121.2625 \\ &= 0.01 \times 1,000 \times 0.021 \times 121.2625 \\ &= 14.99 \mu\text{V}/\text{m} \end{aligned}$$

The leak's field strength is about 15 $\mu\text{V}/\text{m}$, which is below the FCC's 20 $\mu\text{V}/\text{m}$ limit. Technically it's legal, but good engineering practice says if you find a leak, fix it. After all, where there's leakage, there's most likely ingress, too!